Denoising and Baseline Correction of ECG Signals using Sparse Representation

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***Abstract*—Removing noise and other artifacts in the electrocardiogram (ECG) is a critical preprocessing step for further heart disease analysis and diagnosis. In this paper, we propose a sparse representation based ECG signal denoising and baseline wandering (BW) correction algorithm. Unlike the traditional filtering-based methods, like Fourier or Wavelet transform, which use fixed basis, the proposed algorithm models the ECG signal as superposition of few inner structures plus additive random noise, while those structures can be learned from the input signal or a training set. Using those learned inner structures and their properties, we can accurately approximate the original ECG signal and remove noise and other artifacts like baseline wandering. To demonstrate the robustness and efficacy of the proposed algorithm, we compare it to several state-of-the-art algorithms through both simulated and real-life ECG recordings.**

***Index Terms*—sparse representation, adaptive signal separation, ECG denoising, baseline wandering correction.**

I. Introduction

The electrocardiogram (ECG) is a recording of electrical cardiac activity and has been widely applied to the diagnosis of heart disease. As portable devices and electronic medical records have increased the acquisition of ambulatory ECG measures, ECG recordings - which are increasingly longer in duration - are inevitably contaminated by noises and artifacts including additive random noise, baseline wandering (BW) caused by motion and muscle artifact along with ambient electrical fields. Thus, denoising and baseline correction of noisy ECG recordings is a critical technique for practical diagnosis and many related algorithms have been proposed in recent decades.[1], [2]. Traditional filtering approaches were first introduced to remove these contaminations based on their different frequency components. Although these kinds of algorithms can suppress high-frequency noise, they may also distort spike waveforms in the ECG signals because those spike waveforms, like the QRS-complex, usually contain a very wide frequency spectrum.

To overcome drawbacks of standard filtering approaches, some recent advances in adaptive signal processing have been introduced[3], [4], [5], [6], [7], [8], [9]. Since the spike waves in ECG signals are similar to some wavelet bases, some researchers have applied wavelet transform to characterize and locate the waves and have subsequently used thresholding techniques to remove noise[4], [8]. But because wavelet transform cannot effectively remove smooth varying BW, a very narrow low-pass filter was often used to remove BW after the denoising process. Compared to fixed bases projection algorithm, like Fourier and Wavelet transform, Empirical Mode Decomposition (EMD) is a new totally data-driven signal separation approach that can separate a given signal into several mono-components, called Intrinsic Mode Function (IMF). When ECG signal is decomposed by EMD, high frequency noise, ECG waveforms and BW will each be variably distributed to different IMFs. By using this feature, Blanco-Velasco *et al.* proposed a novel EMD-based algorithm which is able to remove both high-frequency noise and BW with small signal distortion[3]. However, because most QRS complex waveforms cannot be fully confined to a single defined IMF, they are usually spread to many IMFs and mixed with noise in the first several IMFs. This phenomenon is called mode-mixing[10]. Combining the advantages of both EMD and Wavelet transform, Kabir and Shahnaz proposed an EMD-Wavelet domain based ECG denoising algorithm[5] which uses wavelet to improve the denoising result in the first several IMFs.

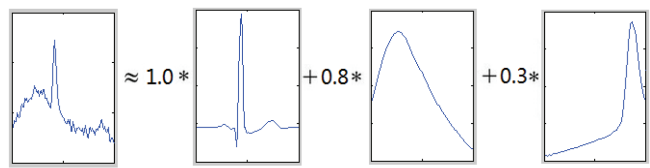

Fig. 1. An example of sparse representation of a segment in ECG recordings.

However, unlike other biomedical signals, ECG signals have its own distinct repetitive patterns which are quite different from random noise and BW. Thus, if we can learn those patterns from the ECG recordings adaptively, they can be represented by these learned patterns (or we may call them inner structures) with great efficiency and effectiveness. To fulfill this task, the signal sparse and redundant representation model[11], [12] comes to our mind. In this model, a redundant dictionary is trained from the input signal or a training set, and

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then, the signal to be analyzed can be sparsely approximated by the atoms in the trained dictionary. For example, as illustrated in Figure 1, given a segment of ECG recordings, we assume that it can be approximated by linear combination of only few atoms in a well-trained dictionary. The residual component of this segment can be viewed as additive random noise. Therefore, our proposed sparse representation based ECG denoising and BW correction algorithm consists of three steps: (1) training a redundant dictionary using the input ECG recording or a training set; (2) using the trained dictionary to represent the ECG recording sparsely to fulfill the signal denoising; and (3) using statistical measures to separate the atoms in the dictionary into two parts; and further, separate the clean ECG signal and BW from the denoised ECG signal. Since the atoms in the dictionary are trained from particular ECG recordings, they can depict the inner structure of ECG recordings. Furthermore, because the difference between ECG patterned waves and BW can also be reflected in the trained atoms, we can remove the BW according to the type of atoms. The experimental comparisons demonstrate that the results derived by our proposed algorithm are better than the results of EMD and Wavelet based thresholding approaches.

The remainder of this paper is organized as follows. In Section II, we review the fast sparse representation algorithm. In Section III, we propose our ECG signal denoising and BW correction algorithm; and in Section IV, we provide some experiments on simulated and real-life ECG signals and discuss their results. Finally, section V contains some concluding remarks.

II. Sparse Representation of Signals

Signal sparse and redundant representation model provides a very effective way to describe the inner structures of signals. For an input signal $\boldsymbol{y} \in \mathbb{R}^n$, this model assumes it can be approximated as $\boldsymbol{y} \approx \boldsymbol{D}\boldsymbol{\alpha}$, where $\boldsymbol{D} \in \mathbb{R}^{n\times m}$ is a dictionary matrix and $\boldsymbol{\alpha} \in \mathbb{R}^m$ is the representation coefficients. Usually, the dictionary $\boldsymbol{D}$ is assumed to be redundant, that is, $m \gg n$; and the coefficients $\boldsymbol{\alpha}$ is assumed to be sparse, i.e., $\|\boldsymbol{\alpha}\|_0$ is very small. The sparsity of $\boldsymbol{\alpha}$ means there are only few non-zero coefficients in $\boldsymbol{\alpha}$, which further implies that the input signal $\boldsymbol{y}$ can be characterized by few atoms (as few columns) from the dictionary matrix $\boldsymbol{D}$. These atoms can be viewed as the description of inner structures of the input signal.

In order to apply this model to practical real-life signals, learning a dictionary that is suitable for a family of signals is the key step. And thus, dictionary learning algorithms have garnered great interest from researchers in the past decade. Given a set of signal examples, $\boldsymbol{Y} = \{\boldsymbol{y}_i\}_{i=1}^N$, dictionary learning algorithms will train a dictionary $\boldsymbol{D}$ which can sparsify and minimize the approximation error of them. This problem can be formulated as

$$\hat{\boldsymbol{D}} = \arg\min_{\boldsymbol{D},\boldsymbol{\alpha}_i} \sum_{i=1}^{N} \|\boldsymbol{y}_i - \boldsymbol{D}\boldsymbol{\alpha}_i\|_2^2, \quad \text{s.t.} \quad \forall i, \|\boldsymbol{\alpha}_i\|_0 < k, \quad (1)$$

where k is the parameter that constrains the maximum number of atoms can be used to approximate the signal. The above optimization problem is very complex because both dictionary and representation coefficients need to be estimated. Most of the existing approaches for solving this problem, like MOD and K-SVD, etc., consist of two steps[12], [11], [13]: (1) given an estimated dictionary, compute the sparse representation coefficients $\boldsymbol{\alpha}_i$; and (2) using the known representation, update the dictionary $\boldsymbol{D}$. This approach will not only provide an approximate solution but also can be highly time consuming because of the slow convergence rate resulting from this alternative optimization style.

To accelerate the solving procedure for the problem represented in Equation (1), Smith and Elad proposed an improved dictionary learning algorithm which uses multiple dictionary updates and coefficients reuse techniques[14]. They formulated the optimization task as

$$\{\hat{\boldsymbol{D}}, \hat{\boldsymbol{A}}\} = \arg\min_{\boldsymbol{D},\boldsymbol{A}} \|\boldsymbol{Y} - \boldsymbol{D}\boldsymbol{A}\|_F^2, \quad \text{s.t.} \quad \boldsymbol{A} \odot \boldsymbol{M} = 0, \quad (2)$$

where $\boldsymbol{A} = [\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_N]$; $\odot$ denotes entry-wise (Schur) multiplication of two equally-sized matrices; and $\boldsymbol{M}$ is a mask matrix of zeros and ones with $M(i,j) = 1$ if $A(i,j) = 0$, and zeros elsewhere. The requirement $\boldsymbol{A} \odot \boldsymbol{M} = 0$ forces all the zero entries in A to remain intact[14]. This constraint optimization problem can also be solved by a block-coordinate-descent approach, that is, for a fixing $\boldsymbol{A}$, $\boldsymbol{D}$ can be updated by

$$\hat{\boldsymbol{D}} = \arg\min_{\boldsymbol{D}} \|\boldsymbol{Y} - \boldsymbol{D}\boldsymbol{A}\|_F^2 = \boldsymbol{Y}\boldsymbol{A}^T \left(\boldsymbol{A}\boldsymbol{A}^T\right)^{-1} = \boldsymbol{Y}\boldsymbol{A}^\dagger, \quad (3)$$

and then, followed by an update of each column in $\boldsymbol{A}$ by fixing $\boldsymbol{D}$ and solving

$$\hat{\boldsymbol{\alpha}}_i = \arg\min_{\boldsymbol{\alpha}_i} \|\boldsymbol{y}_i - \boldsymbol{D}_i\boldsymbol{\alpha}_i\|_2^2 = \boldsymbol{D}_i^\dagger \boldsymbol{y}_i, \quad (4)$$

where $\boldsymbol{D}_i$ is a sub-matrix of $\boldsymbol{D}$ containing only the atoms in the support of this representation. When all the representation $\boldsymbol{A}$ is derived, the mask matrix $\boldsymbol{M}$ will be updated according to the representation $\boldsymbol{A}$. Although this alternative iteration style is similar to those used in MOD and K-SVD, it is much faster because of the constraint mask matrix $\boldsymbol{M}$. For more detailed discussions and implementations of this improved dictionary learning algorithm, readers may refer to the reference [14].

III. Baseline Correction and Denoising of ECG Signals

In this section, we intend to discuss the approach for simultaneous denoising and baseline correction of ECG signals. As illustrated in the introduction part, typical ECG signals contain P-wave, QRS complex and T-wave, which means ECG signals can be sparsely represented by those inner structures. Meanwhile, the smooth varying BW can also be represented by those harmonic atoms, like DCT-bases or Gabor atoms. But for random noise, to the authors' knowledge, there exists no dictionary or bases that are fully depictive or representative. As a consequence, our ECG signal enhancement algorithm is comprised of three steps: (1) using the given ECG signals to generate a training set and then learning a dictionary

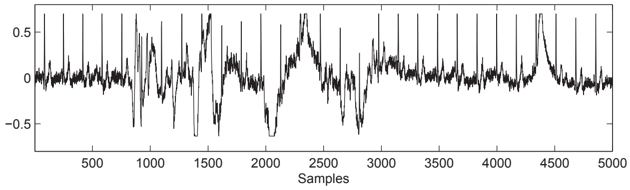

Fig. 2. An ECG recording s00377 from PhysioNet MIMIC II Waveform Database Matched Subset.

that fits for this signal; (2) using the trained dictionary to derive a sparse representation of the given ECG signal; and (3) removing the noise by using the sparse coefficients to reconstruct the signal; and furthermore, separating the ECG signal and BW via dividing the atoms in the dictionary into two subsets according to their statistical characteristics.

A. Dictionary Training of ECG Signals

Since our algorithm is based on signal's sparse and redundant representation, the input signals will be processed by an overlapped segment-by-segment style. That is, for an input signal $s(t)$ with length L, if we define the length of a segment is l, $s(t)$ will be divided into many segments as $s_i(t) = [s(i), s(i+1), \cdots, s(i+l-1)]$ with $i = 1, \tau+1, 2\tau+1, \cdots, L-l+1$ where $\tau < l$ is denoted as the time shift; and then, each segment $s_i(t)$ will be processed or reconstructed as $\hat{s}_i(t)$, successively. Then, the processed or reconstructed signal $\hat{s}(t)$ can be calculated via a weighted average of all the processed segments as

$$\hat{s}(t) = \frac{1}{n+1} \sum_{k=0}^{n} \hat{s}_{i+k\tau}(t-i-k\tau), \text{ with } i < t < i+n\tau. \quad (5)$$

And thus, if the length of segments is chosen as l, the size of dictionary $\boldsymbol{D}$ should be $l \times m$ with $m \gg l$; and the number of segments in the training set should be much larger than m.

Theoretically, we can randomly select enough number of segments from some standard ECG signal database to learn a universal dictionary for processing all input ECG signals. However, ECG recordings often contain a significant amount of diversity stemming from differing lead placements, inter-individual variability and conduction pathologies. For this reason, there is no single, truly "typical" ECG waves, and thus, using a standard, universal dictionary to process all variations of ECG signals will not derive a satisfactory result. Furthermore, because of the improved dictionary algorithm[14] as described in section II, the time cost for training process has been sped up greatly and an individual, case-by-case learning approach becomes feasible. Therefore, in this paper, for a given ECG signal, we will train a dictionary using its own waveform as reference. Although ECG signals recorded from different people may have different wave patterns, wave patterns in a given ECG signal are similar and much more representative. Furthermore, for a long term ECG recording, partitioned segments can be randomly selected to efficiently train the dictionary rather than using the whole signal. For our

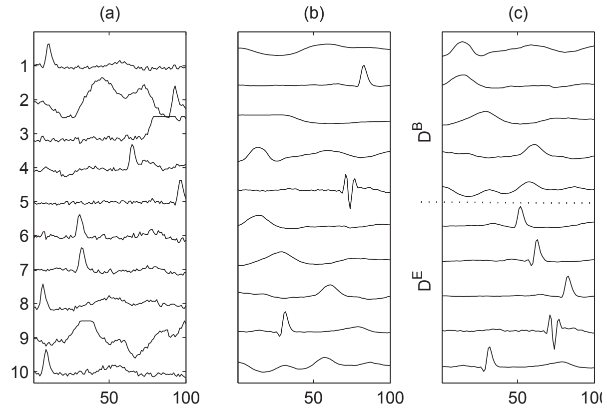

Fig. 3. Examples of training set and learned dictionary: (a) some examples of raw signal segments in the training set; (b) some atoms in the dictionary learned from the training set; (c) some atoms in the separated sub-dictionaries (as $\boldsymbol{D}^B$ and $\boldsymbol{D}^E$).

proposed dictionary training on a given ECG signal (as shown in Figure 2), we randomly choose segments with 50 times the number of atoms in the dictionary to build our training set. Some of the segment examples can be found in Figure 3(a). We subsequently use the improved dictionary learning algorithm[1] to further train a dictionary via this training set. Figure 3(b) shows some of the atoms in the trained dictionary of this ECG recording, from which we find that some of the atoms reveal distinguishing features of the ECG signal.

B. Synchronous Denoising and Baseline Correction

For a given ECG recording $s(t)$, we assume it contains three component signals, a clean ECG signal $s^E(t)$, BW artifacts $s^B(t)$, and additive random noises. An ECG recording can thus be modeled as the superposition of these components as

$$s(t) = s^E(t) + s^B(t) + \text{noise}. \quad (6)$$

Then, with already trained dictionary $\boldsymbol{D}$, the input signal $s(t)$ will be processed segment-by-segment as described in the above subsection. For a segment signal $\boldsymbol{s}_i = s_i(t)$, we use Orthogonal Matching Pursuit (OMP) algorithm[15] to compute its sparse representation of dictionary $\boldsymbol{D}$ denoted as $\boldsymbol{\alpha}_i$. Since the dictionary $\boldsymbol{D}$ is trained from this recording itself, it can effectively and sparsely represent the ECG signal and BW while excluding random noises. That is, we can assume that the reconstructed segment $\hat{\boldsymbol{s}}_i$ can keep most of useful informations in the original segment as

$$\hat{\boldsymbol{s}}_i = \boldsymbol{D}\boldsymbol{\alpha}_i \approx \boldsymbol{s}_i^E + \boldsymbol{s}_i^B. \quad (7)$$

Meanwhile, because ECG signals usually have sharp peaks and BW are smooth varying, this significant difference should also be reflected in the learned dictionary (as shown in Figure 3(b)). So, we compute the kurtosis of each atom $\boldsymbol{d}_j$ (denoted as $\kappa(\boldsymbol{d}_j)$) in the dictionary $\boldsymbol{D}$ and derive its statistical histogram

[1]This algorithm is described briefly in section II, and its source codes can be downloaded from http://www.cs.technion.ac.il/~elad/Various/ImprovedDL.rar

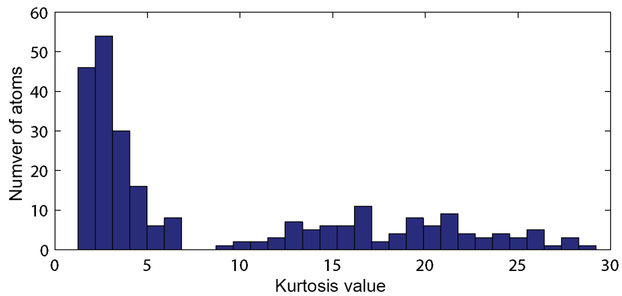

Fig. 4. Statistical histogram of kurtosis of all the atoms in the learned dictionary $\boldsymbol{D}$.

Algorithm 1 ECG Denoising and Baseline Correction Algorithm

1: Input ECG recording $\boldsymbol{s}$, choose the length l for each segment, the number of atoms in the dictionary as n, and a threshold λ.
2: Randomly select $50n$ of segments from $\boldsymbol{s}$ to build the training set;
3: Learn a redundant dictionary $\boldsymbol{D} \in \mathbb{R}^{l \times n}$ from the training set;
4: Compute kurtosis of each atom $\boldsymbol{d}_j \in \boldsymbol{D}$ denoted as $\kappa(\boldsymbol{d}_j)$;
5: Divide the dictionary into two sub-dictionary with $\boldsymbol{D}^E = \{\boldsymbol{d}_j; \kappa(\boldsymbol{d}_j) > \lambda\}$ and $\boldsymbol{D}^B = \{\boldsymbol{d}_j; \kappa(\boldsymbol{d}_j) \leq \lambda\}$;
6: **for** each segments $\boldsymbol{s}_i$ in $\boldsymbol{s}$ **do**
7: Compute its sparse representation coefficient $\boldsymbol{\alpha}_i$ with dictionary $\boldsymbol{D}$;
8: Separate the coefficients into two parts as $\boldsymbol{\alpha}_i^E$ and $\boldsymbol{\alpha}_i^B$ according to the sub-dictionary $\boldsymbol{D}^E$ and $\boldsymbol{D}^B$;
9: Reconstruct the clean ECG and BW segment as $\hat{\boldsymbol{s}}_i^E = \boldsymbol{D}^E\boldsymbol{\alpha}_i^E$ and $\hat{\boldsymbol{s}}_i^B = \boldsymbol{D}^B\boldsymbol{\alpha}_i^B$, respectively;
10: **end for**
11: **return** Reconstruct the clean ECG signal $\hat{s}^E(t)$ by weighted average of all the segments $\hat{\boldsymbol{s}}_i^E$;

as shown in Figure 4. Given a threshold parameter λ, the atoms in $\boldsymbol{D}$ can be clearly divided into two sub-dictionaries, denoted as $\boldsymbol{D}^E$ and $\boldsymbol{D}^B$, with $\boldsymbol{D}^E = \{\boldsymbol{d}_j; \kappa(\boldsymbol{d}_j) > \lambda\}$ and $\boldsymbol{D}^B = \{\boldsymbol{d}_j; \kappa(\boldsymbol{d}_j) \leq \lambda\}$. The sub-dictionary $\boldsymbol{D}^E$ contains atoms with sharp peaks and have large kurtosis values; while the sub-dictionary $\boldsymbol{D}^B$ includes smooth varying atoms and have small kurtosis values. Some atoms in these separated two sub-dictionaries can be found in Figure 3(c). Furthermore, with these two sub-dictionaries, we can also divide the sparse coefficients $\boldsymbol{\alpha}_i$ of each segment into two parts, as $\boldsymbol{\alpha}_i^E$ and $\boldsymbol{\alpha}_i^B$. And the estimated ECG signal $\hat{s}^E(t)$ can be derived by weighted average of all these ECG segments $\hat{\boldsymbol{s}}_i^E = \boldsymbol{D}^E\boldsymbol{\alpha}_i^E$. The overall ECG signal denoising and baseline correction algorithm can be summarized as in Algorithm 1

Some Implementation Details. In Algorithm 1, the length l for each signal segment depends on the sampling rate of ECG recordings. Since a single heart beat containing intact P-wave, QRS complex and T-wave is usually less than one second, if the sampling rate is PHz, we set the length of signal segment to be $0.8P$ samples in all of our experiments. We set the number of atoms in the dictionary to be $2P$ because it should be larger than the length of segment to ensure that the learned dictionary is redundant. The other parameter to be considered is the maximum number of atoms used to reconstruct the signal which reflects the sparseness of the representation. Through our own preliminary tests, we have found that this approach is quite robust for reconstructing noisy ECG signals. For this reason, we set the maximum number of atoms to be $0.1P$. The last parameter to consider is the threshold λ for dividing the learned dictionary into two sub-dictionaries. As can be seen from Figure 4, the value of λ can be estimated from the statistical histogram. However, due to the risk of the training process failing to converge towards a satisfactory dictionary, we use a fixed value ($\lambda = 6$) in all the following experiments.

IV. Experimental Results

In this section, we compare our proposed algorithm with other state-of-the-art ECG denoising algorithms in experiments using real-life and simulated ECG signals. The methods used for comparison include EMD soft-thresholding algorithm[3] (denotes EMD for short) and Wavelet thresholding algorithm[8] (denotes DWT for short). The ECG recordings used for experiments come from MIMIC II Waveform Database Matched Subset in PhysioNet [16].

In **Real-life ECG experiment 1**, we use our algorithm to denoise and correct baseline of ECG recording in Figure 2. Since the original ECG recording contains several hours of data, we only show 5000 samples (approximately 39 seconds) of this signal in Figure 2, . Some atoms in the trained dictionary can be found in Figure 3(c). The denoised and baseline corrected ECG signal and the removed BW are shown in Figure 5(a) and 5(b), respectively. From which, we can find that the BW is clearly removed from the original input ECG signal. To analyze the result in more detail, we zoom two segments in Figure 5(a), as from samples 1501-2500 and 4001-5000, and plot them in Figure 5(c) and 5(d), respectively. In each sub-figure, black and red lines denote the input ECG signal and the denoised result of our proposed algorithm, respectively. We can observe that our algorithm effectively isolates and preserve the P-waves, QRS complexes, and T-waves in the denoised ECG signal, despite the presence of strong noise and large varying BW in the original signal.

In **Real-life ECG experiment 2**, we compare the ECG denoising and baseline correction results derived by our proposed algorithm, EMD algorithm and DWT algorithm. The input ECG recording is shown in Figure 6(a), from which we can find that its ECG waves are different from standard ECG waves. Figures 6(b) to (d) show the results derived from our algorithm, EMD and DWT approaches, respectively. Because the input signal has minimal noise, all algorithms tested in this experiment perform well in denoising and correcting the baseline. But as made evident by closer investigation of the plots in Figure 6(e), EMD-based algorithm will generate obvious over shock adjacent to the QRS complex. This may

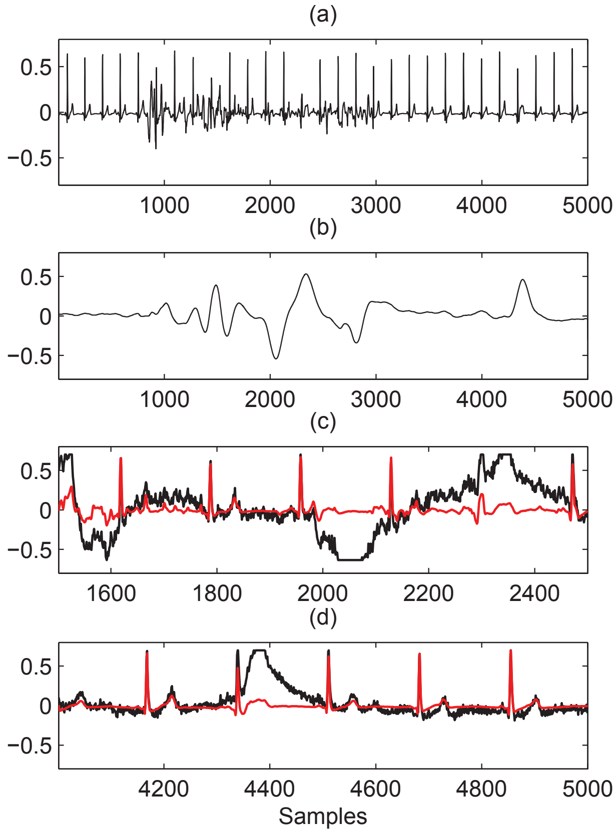

Fig. 5. Denoised and baseline corrected ECG recording in Figure 2. (a) the denoised and baseline corrected ECG recording; (b) the removed baseline wander; (c) and (d) the zoomed detailed ECG waves in (a) from samples 1501 to 2500 and 4001 to 5000, respectively. In (c) and (d), the black line and red line denotes the original ECG recording in Figure 2 and the denoised and corrected ECG recording in (a), respectively.

TABLE I
SNR IMPROVEMENT AND RMSE OF SIMULATED EXPERIMENTS.

Methods / Input (SNR/RMSE)	SNR_{imp}/RMSE		
	EMD Threshod	DWT Threshold	Proposed Algorithm
10dB / 0.281	7.92 / 0.113	9.04 / 0.098	**13.45 / 0.059**
15dB / 0.193	9.17 / 0.067	11.13 / 0.052	**17.21 / 0.026**
20dB / 0.160	11.54 / 0.045	16.49 / 0.036	**21.22 / 0.013**

arise from the asymmetry of this QRS complex which departs from the definition of IMF, defined as the mono-component signal in the EMD algorithm. Furthermore, the DWT-based algorithm does not effectively remove the BW from the input signal.

In the last experiment, we use a **Simulated ECG Signal** to quantitatively compare the performance of our proposed algorithm and the other two algorithms. The clean ECG signal is the recording s00052 in the MIMIC II Matched Subset. Since the whole ECG recording contains several hours of data, we selected only 10 clean segments, each with 2000 samples. For each segment, we subsequently added Gaussian white noise with differing SNR values (generated by MatLab function *awgn*) and a baseline wander (generated by $\cos(2\pi t)$).

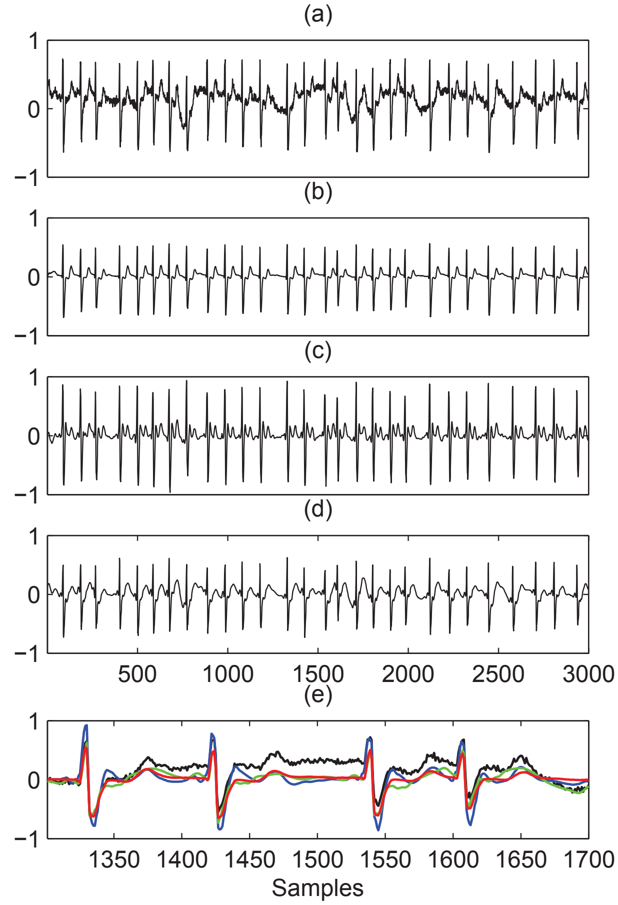

Fig. 6. Denoised and baseline corrected ECG recording s00262 in MIMIC II Matched Subset. (a) the input ECG recording with 3000 samples; (b), (c), and (d) the denoised and baseline corrected ECG signals derived by our proposed algorithm, EMD soft thresholding algorithm[3] and Wavelet thresholding algorithm[8], respectively; (e) the zoomed details of derived ECG waves from samples 1301 to 1700. The black, red, green, and blue lines denote the original ECG recording, the result of our proposed algorithm, the Wavelet thresholding algorithm and EMD thresholding algorithm, respectively.

We then used different algorithms to remove BW and noise from each simulated signals to derive the output signal. Finally, we compare the mean performance of these 10 signals based on two metrics: improvement in signal-to-noise-rate (SNR_{imp}) and the Root of Mean Square Error (RMSE), defined as:

$$\text{SNR}_{imp} = 10 \log_{10} \frac{\sum_{i=1}^{n} (y[i] - x[i])^2}{\sum_{i=1}^{n} (\hat{x}[i] - x[i])^2}, \tag{8}$$

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{x}[i] - x[i])^2}. \tag{9}$$

As in Equation (8) and (9), the higher SNR_{imp} value and lower RMSE value represent better results.

In simulated signals, the Signal-to-Noise Ration (SNR) parameters in the MatLab function *awgn* were set to SNR values of 10dB, 15dB, and 20dB. Table I presents the comparison of these two metrics between our proposed algorithm, EMD-based algorithm and DWT-based algorithm for the same group of simulated ECG signals. The results indicate that our

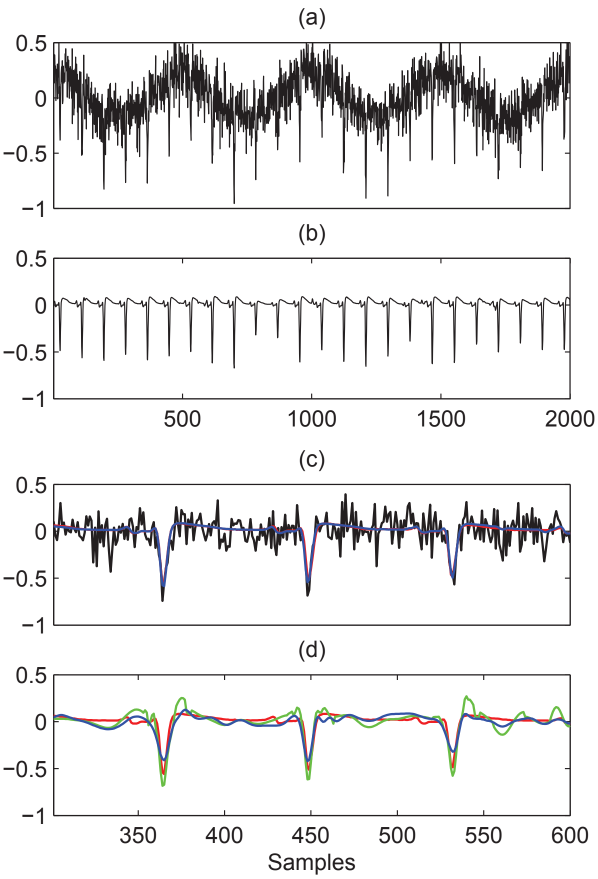

Fig. 7. Simulated denoising and baseline correction experiment. (a) The simulated noisy and baseline wandering signal with SNR $= 15$dB. (b) The denoised and baseline corrected signal derived by our proposed algorithm. (c) The detailed comparison between noisy signal (in black line), real clean signal (in red line) and the denoised signal derived by our approach (in blue line). (d) The detailed comparison between real clean signal (in red line), results of EMD thresholding (in green line) and DWT thresholding (in blue line).

algorithm outperforms the other two algorithms. We illustrate a sample simulated noisy signal (with SNR $= 15$dB) in Figure 7(a), while Figure 7(b) reveals the denoised and baseline corrected result derived by our proposed algorithm. In Figure 7(c) and (d), detailed comparisons between the real clean signal, our result and the results of EMD and DWT algorithms can be found. The details can clearly reflect that our derived denoised signal more closely resemble the original clean signal compared to the signals derived by the other two algorithms.

V. Conclusion and Discussion

In this paper, a sparse representation based ECG signal denoising and baseline correction approach is presented. Signal sparse and redundant representation is a recent new signal processing method which can effectively learn the inner structure from noisy input signals. Moreover, the characteristics P-, T- waves and QRS complexes in the ECG recordings and smooth varying baseline wandering are revealed in the learned dictionaries. We use the learned dictionary to denoise and effectively reconstruct the ECG signal using selected atoms in the dictionary and remove the baseline wandering. Also, the real-life and simulated experimental recordings demonstrate that our approach can effectively remove the noisy and baseline wandering in the ECG signals while maintaining the intrinsic ECG waves. Since our proposed algorithm is nearly parameter-free , it is likely amenable to automized ECG analysis systems. The general implementation of this algorithm to such systems and to clinical situations (with varying pathologies) warrants further study.

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